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The Oxford Calculatores, *Quantification of Qualities*, *and Aristotle's Prohibition of* Metabasis

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When Walter de Merton founded Merton College in 1264, he sought to provide a residence hall for Arts students intending to proceed to the Faculty of Theology at Oxford. Nevertheless, with the exception of Thomas Bradwardine, Merton scholars from the first half of the fourteenth century have achieved widespread fame not for their achievements in sacred theology, but rather for their contributions in the fields of natural philosophy and mathematics. The precise nature of the Mertonian achievement is a topic which has been discussed at great length during the past seventy-five years,[1] and a subject into which this paper will not venture. Instead, its focus will be Mertonian methodology and in particular the relationship between the Mertonians' work and a problem first discussed by Aristotle in his *Posterior Analytics*.

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[1] Regarding the institutional development of Merton College, see Hastings Rashdall, *The Universities of Europe in the Middle Ages*, ed. F. M. Powicke and A. B. Emden, 3 vols., Oxford 1936, vol. 3, 191-201. One can scarcely begin to summarize the literature on the Mertonian work. See in particular the five volumes of Anneliese Maier's *Studien zur Naturphilosophie der Spätscholastik*, Rome 1949-1958; Marshall Clagett, *The Science of Mechanics in the Middle Ages*, Madison, Wisc. 1959; Curtis Wilson, *William Heytesbury: Medieval Logic and the Rise of Mathematical Physics*, Madison, Wisc. 1956; A. G. Molland, *The Geometrical Background to the 'Merton School'*, in: British Journal for the History of Science, 4 (1968), 108-125; and two excellent dissertations, James A. Weisheipl, *Early Fourteenth-Century Physics of the Merton School with Special Reference to Dumbleton and Heytesbury*, Oxford 1956, and Edith D. Sylla, *The Oxford Calculators and the Mathematics of Motion, 1320-1350: Physics and Measurement by Latitudes*, Harvard 1970. See also Sylla's subsequent articles, *Medieval Quantification of Qualities: the 'Merton School'*, in: Archive for History of Exact Sciences, 8 (1971), 9-39 and *Medieval Concepts of Latitudes of Forms: the Oxford Calculators*, in: Archives d'histoire doctrinale et littéraire du moyen âge, 40 (1973), 223-283. Sylla and other recent scholars have argued that the appellation "Mertonian" is incorrect because fourteenth-century work generally discussed as Mertonian was in fact not restricted to Merton College masters. In what follows, I have used the traditional name *large* rather than *stricte*.

While the *Posterior Analytics* as a whole contains Aristotle's thoughts on demonstrative science, in Book I, chapter 7 he discusses a particular aspect of the method known as *metabasis*, that is, the transition from one scientific discipline to another in the process of demonstration. For a variety of reasons, both ontological and methodological in nature,[2] Aristotle argues that science can be organized into distinct categories or disciplines and that it is impermissible to prove propositions by passing from one such genus to another (οὐκ ... ἔστιν ἐξ ἄλλου γένους μεταβάντα δεῖξαι). As is often the case in the *Posterior Analytics*, Aristotle produces a mathematical example to reinforce his point: it is not possible, he says,

> to prove a geometrical proposition by arithmetic. ... Where different genera, e.g. arithmetic and geometry, are involved, although the basis of the proof may be the same, it is not possible to apply the arithmetical demonstration to the attributes of extended magnitudes, unless magnitudes are numbers.[3]

Nevertheless, Aristotle was compelled almost from the beginning to modify his prohibition somewhat, for in chapter 7 and again in chapters 9 and 13 he admits that the so-called subalternating and subalternate sciences—astronomy, music, optics, and the like—constituted a small group of exceptions to his general rule.[4]

When the *Posterior Analytics* entered the Latin West in the middle of the twelfth century,[5] Aristotle's prohibition of *metabasis* generated

[2] Aristotle's rejection of *metabasis* arises in part from his criticism of the Pythagoreans and Platonists. In *Metaphysics* VII.11 $1036^{b}8$-21, for example, he rejects the reduction (ἀναγωγή) of all things to numbers, not simply because he feels numbers are not the proper candidates, but because more fundamentally he opposes the reduction of all things to one genus. More explicitly, in *Metaphysics* IV.2 $1004^{a}4$ and I.9 $992^{b}19$-$933^{a}7$ he rejects any notion of a Platonic master-science to which all the other sciences are reducible because there is no genus which encompasses all things. Methodologically, Aristotle says that the existence of such a master-science would imply that the first principles of the sciences beneath it would all be proven by the master science, and hence all syllogisms would by reduction have the same first principles. Aristotle rejects this emphatically in *Posterior Analytics* I.32 $88^{a}19$. Finally, the ability to transcend scientific genera was a property Aristotle recognized in Platonic dialectic, and against which he developed his own theory of demonstration. For a discussion of this important issue, see below, p. 62.

[3] *Posterior Analytics* I.7 $75^{a}38$-$^{b}6$.

[4] *Posterior Analytics* I.7 $75^{b}15$; I.13 $78^{b}35$-$79^{a}17$. That Aristotle saw the subalternate sciences as a relatively constricted exception is suggested by his remarks in *Post. An.* I.9 $76^{a}10$ and $76^{a}23$-25, εἰ δὲ μή, ἀλλ' ὡς τὰ ἁρμονικὰ δι' ἀριθμητικῆς. Regarding Aristotle's theory of the subalternate sciences, see Richard D. McKirahan, Jr., *Aristotle's Subordinate Sciences*, in: British Journal for the History of Science, 11 (1978), 197-220.

[5] Regarding the Latin transmission of the *Posterior Analytics*, see Charles H. Haskins, *Mediaeval Versions of the Posterior Analytics*, in: *Studies in the History of Mediaeval Science*, 2nd

significant interest among scholars studying the method of demonstrative science. The opening lines of chapter 7 became a frequent *lemma* for literal commentaries, and because they likewise contained the most explicit account of Aristotle's prohibition, virtually every commentary on the *Analytics* discussed the nature of Aristotle's remarks. Similarly, when questionaries became the popular genre of commentary later in the thirteenth century, such queries as "Utrum contingat ex alio genere descendentem in aliud genus demonstrare?" or "Utrum possibile est demonstratione descendere de genere in genus?" became standard features of the commentary tradition.[6]

The new directions in natural philosophy taken early in the fourteenth century rested on, among other things, the quantification of qualities, a procedure whose justification modern scholars have found problematic for a variety of reasons, including the disjunction of quantity and quality in the categories[7] and the tension between mathematical exactness and the obvious inexactness of the physical world.[8] But to those scholars working within the university tradition of the *Posterior Analytics*, Aristotle's prohibition of *metabasis* would constitute still another, disciplinary obstacle to any attempt to quantify qualities or inject mathematics into the domains of natural philosophy or other sciences. It becomes, therefore, a significant question to ask how Mertonians and their like-minded Oxonians undertaking such a procedure could also reconcile their work with Aristotle's prohibition of *metabasis*.

Attempts to understand Mertonian positions on the prohibition of *metabasis* are made difficult by the fact that taken as a group, the Mertonians produced virtually no commentaries on the *Posterior Analytics*.

edn., Cambridge, Mass. 1927, 223-241 and more recently, *Aristoteles Latinus IV.1-4: Analytica Posteriora*, ed. L. Minio Paluello and B. Dod, Bruges-Paris 1968, ix-lxxxiii. Several articles in Minio-Paluello's *Opuscula* (Amsterdam 1972) also bear on the topic, including *L'ignota versione moerbekana dei Secondi Analitici usata da S. Tomaso* (pp. 155-163) and *Iacobus Veneticus Grecus: Canonist and Translator of Aristotle* (pp. 189-228).

[6] Medieval scholars' concerns about *metabasis* may not have originated exclusively in logical investigations. Robert Kilwardby, for example, spends some five folios of his commentary on the *Posterior Analytics* discussing *metabasis* and the related issue of the subalternate sciences (Cambridge, Peterhouse 205, fol. 142^{v}-147^{r}), but somewhat later, in the *De ortu scientiarum* ch. 16, section 96, his interest in *metabasis* is clearly tied to the problem of the classification of the sciences and the relationship between one science and another. See *De ortu scientiarum*, ed. Albert G. Judy, London 1975, 42.

[7] See, for example, Edith Sylla, *Medieval Quantification of Qualities* (note 1 above), p. 9.

[8] A. G. Molland, *An Examination of Bradwardine's Geometry*, in: Archive for History of Exact Science, 19 (1978), 113-175 at 131-138.

Early members of the college, like Thomas Wilton and John Maudith did not write one, nor did Thomas Bradwardine.[9] According to John Bale, Robert Karew (or Cary), who was a fellow at Merton between 1326 and 1332, produced a questionary on the *Analytics* in 1325, a work that once was in the Oxford University Library but seems no longer extant.[10] Various other commentaries on the *Posterior Analytics* once attributed to early Mertonians must also be treated as questionable. Bale again remains the only reference to a commentary by Thomas Sutton (d. 1311/15, and perhaps a Fellow between 1263 and 1274).[11] A *Quaestiones super libros posteriorum* preserved in Gonville and Caius MS 512 is attributed to Petrus de Insula, but whether this work was produced by the Petrus who was a fellow at Merton between 1284 and 1307 or three other candidates remains uncertain.[12] More importantly and certainly, the generation of the 1330s and 40s—Heytesbury, Dumbleton, Swineshead, and their lesser-known colleagues William Sutton, Richard Billingham, Simon Bredon, and Thomas Buckingham—left no record of having written commentaries on the work.[13]

If we cast our net somewhat wider and compare Oxford's production with that of the University of Paris in the second half of the thirteenth century and all of the fourteenth century, we find a similar situation. John Murdoch has noted recently that for the period 1200-1330, commentaries on the *Physics* produced at Oxford outnumbered those produced at Paris by a margin of better than two to one and suggested that this may be partially related to the peculiar

[9] James A. Weisheipl, *Repertorium Mertonense*, in: Mediaeval Studies, 31 (1969), 174-224 at 177-182, 217-218, 222-224.

[10] John Bale, *Index Britanniae Scriptorum*, ed. R. L. Poole and Mary Bateson, Oxford 1902, 382. Concerning Karew, see A. B. Emden, *A Biographical Register of the University of Oxford to A.D. 1500*, 3 vols., Oxford, 1957-1959, vol. 1, 366-367.

[11] Bale, 455. See also Emden, vol. 3, 1824-1825.

[12] Concerning the four candidates, see Charles H. Lohr, *Medieval Latin Aristotle Commentaries, Authors Narcissus-Richardus*, in: Traditio, 28 (1972), 281-396 at 362. Concerning the text at fols. 57-87^{v}, see M. R. James, *A Descriptive Catalogue of the Manuscripts in the Library of Gonville and Caius College*, 2 vols., Cambridge, 1907-1908, vol. 2, 581-584, where the text is attributed to Will(iam) Dallinge at fol. 57 and Petrus de Insula at fol. 81.

[13] See Weisheipl (note 9 above). One should note, however, that Bredon's will refers to "quaternos meos de grammatica et dialectica;" F. M. Powicke, *The Medieval Books of Merton College*, Oxford 1931, 84. Bale [(note 10 above) 411] refers to a commentary on logic assigned to Bredon "seu Byridanus" that he says he saw in the Magdalen College library; Poole links this to MS 88, which contains an anonymous logical commentary assigned conjecturally to Buridan.

character of Oxonian calculatory natural philosophy.[14] Precisely the opposite seems to be the case for commentaries on the *Posterior Analytics*, with Paris outproducing Oxford by a factor of two to one. This, combined with the dearth of commentaries at Merton in particular is in itself an interesting state of affairs, because the *Posterior Analytics* could not have been unknown to Oxford scholars in general and Mertonians in particular. On the one hand, the *Analytics* was required of all Arts students at Oxford, as the statutes of the University clearly specify.[15] And closer to the College itself, the records of the Merton Library indicate that the text of the *Posterior Analytics* was available in four manuscripts present in the library before 1325, another added between 1325 and 1360, and several which are only mentioned in the *electiones* or lending lists of 1372 and could have been present earlier in the century.[16] In addition, the library also acquired commentaries on the *Analytics*, including three copies of the commentary by Robert Grosseteste—one of which entered the library perhaps between 1325 and 1346—, two by Thomas Aquinas, and one by Simon of Faversham.[17] And while we do not know the extent to which

[14] John E. Murdoch, *The Analytic Character of Late Medieval Learning: Natural Philosophy without Nature*, in: *Approaches to Nature in the Middle Ages*, ed. Lawrence D. Roberts, Binghamton, NY 1982, 171-213 at 197. My statistics have been derived from a tabulation of commentaries listed in Charles Lohr's *Medieval Latin Aristotle Commentaries*, in: Traditio, 23 (1967) 313-413; 24 (1968) 149-245; 26 (1970) 135-216; 27 (1971) 251-351; 28 (1972) 281-396; 29 (1973) 93-197; and 30 (1974) 119-144. Against 36 commentaries on the *Posterior Analytics* produced at Paris between 1250 and 1400, one finds only 16 produced at Oxford.

[15] *Statuta antiqua universitatis Oxoniensis*, ed. Strickland Gibson, Oxford 1931, 26: "...quod omnes libros veteris logicae ad minus bis audierint, exceptis libris Boecii, quos semel sufficiat audiuisse preter quartum librum *Topicorum* Boecii, quem audiuisse non astringantur. De nova autem logica librum Priorum, Topicorum, Elenchorum, bis, librum autem Posteriorum, saltem una vice iurent se audiuisse. ... Si autem de sophismatibus publice non responderint, omnes libros predictos iurent se audisse, hoc adiecto, quod bis audierint librum Posteriorum." Concerning the medieval curriculum at Oxford, see James A. Weisheipl, *Curriculum of the Faculty of Arts at Oxford in the Early Fourteenth Century*, in: Mediaeval Studies, 26 (1964), 143-185. One should, however, be cautious in giving too much weight to university statutes as a reflection of actual curricular practice, as Mordicai Feingold has recently emphasized for late sixteenth- and early seventeenth-century English universities; see his *Science, Universities, and Society in England, 1580-1640* (Ph.D. dissertation, Oxford 1981).

[16] For those in the library before 1325, see Powicke, *The Medieval Books* (note 13 above) no. 32, 48, 62, and 63; between 1325 and 1360, no. 104; and for those included among the *electio* of 1372 (nos. 393-402), no. 393, 397, and 401 specifically include the *logica nova*.

[17] Grosseteste's commentary is included in Powicke no. 207 (= Merton College MS 280), which entered the Library perhaps between 1325 and 1345, no. 515 (= Merton MS 289), acquired before 1385, and no. 518 (= Merton MS 292), also acquired before

these were read in the first half of the century, the first lending list available—dating from 1372[18]—indicates that both the texts and the commentaries on the *Posterior Analytics* were popular in the second half of the fourteenth century. In view of the fact that the *Posterior Analytics* generated increasing numbers of commentaries elsewhere in the fourteenth century, it seems rather odd that the Mertonians of the first half of the century should have left no commentary record of the text.

The exception to this observation is Walter Burley, who was a fellow at Merton by 1305, but who spent much of his career on the Continent, first at Paris and then at Avignon.[19] Burley seems to have composed not one, but perhaps three commentaries on the *Posterior Analytics*.[20] Nevertheless, Burley—like Aristotle—generally argues disciplinary autonomy among the sciences. Paraphrasing Aristotle, he notes that arithmetic and geometry have some common principles such as the axiom that equals subtracted from equals yield equals, yet diverse sciences such as arithmetic and geometry do not have the same subject genus or the same principles of the subject genus, and therefore a transference from one genus to another is impossible.[21] But Burley, again like Aristotle, also suggests that there is a legitimate means of descending from one genus into another on certain occasions, and he divides these into two cases. On the one hand, one may

1385. Aquinas' commentary is contained in no. 126 (= Merton MS 296) acquired before 1360 and also in no. 357 (= Merton MS 277) acquired before 1385. Faversham's commentary is contained in no. 518. The availability of Grosseteste's work is interesting, considering his discussion of partial subordination of the sciences which seems to have stood behind Ockham's idea discussed below, pp. 57-58.

[18] See the text in Powicke, pp. 60-63.

[19] Concerning Burley's career, see C. Martin, *Walter Burley*, in: *Oxford Studies Presented to Daniel Callus*, Oxford 1964, 194-230.

[20] Weisheipl, *Repertorium* (note 9 above) attributed three commentaries to him, including one literal commentary, a second abbreviated version printed in the Venice, 1514 edition of Grosseteste's commentary, and a questionary on the *Posterior Analytics* in Gonville and Caius 668*/645.

[21] *Expositio super libros duos Posteriorum*, London, Lambeth Palace MS 70, fol. 154va: "...ita enim est una dignitas communis geometrie et arismetice si ab aequalibus aequalia demas et cetera. Sed diverse scientie non habent idem genus subiectum nec eandem passionem generis subiecti ut arismetica et geometria, et ideo non convenit descendere a geometria in arismeticam quia non habent idem genus subiectum. Non enim habent idem genus subiectum nisi magnitudines et numeris essent idem." The last phrase, paraphrased from Aristotle, occupied the attention of medieval commentators beginning with Grosseteste; rather than taking the phrase in the sense of a contrary to fact statement, as Aristotle likely had intended it, most commentators sought to explain how magnitudes might be numbers, and proposed a theory of *magnitudo numerata* and *non-numerata*. Burley, however, does not elaborate on this material.

transfer a demonstration when the subject genus of one simply constitutes a species of the other; this is trivially true, says Burley, because the same science that examines triangles also examines isosceles triangles. On the other hand, demonstrations may be transferred when the genera are the same "in a certain way" (*quodammodo*). This occurs, according to Burley, among the subalternate sciences when, for example, a proof involving a line is used to prove a property involving visual lines.[22] Burley's analysis of *metabasis*, in other words, is congruent with that of Aristotle, and hence he is of little help in determining a Mertonian response to the problem of *metabasis* and the techniques of the next generation of work.

Despite this setback, there are several aspects of Mertonian or at least early fourteenth-century Oxonian work which suggest responses to the problem of *metabasis*. The first concerns the subalternating and subalternate sciences or *scientiae mediae* as they were known in the Middle Ages. As we have seen already, Aristotle himself recognized exceptions to his prohibition of *metabasis*: the sciences of astronomy, optics, music and the like prove their propositions by recourse to principles taken from the higher sciences of arithmetic and geometry. Based upon his discussion in the first book of the *Analytics*, Aristotle seems to have considered this a rather narrow exception to the general rule of disciplinary autonomy,[23] yet there is evidence that would suggest that at least by the opening years of the fourteenth century, medieval scholars were beginning to adopt a more liberal version of this rule. And while he is neither alone nor the first in this movement, I should like to illustrate briefly the lengths to which this shift was taken by a non-Mertonian whose logical and philosophical orientation was frequently adopted by later Mertonians, William of Ockham.[24]

[22] *Expositio*, fol. 154va: "...potest superius descendere in inferiora dupliciter: uno modo per differentiam essentialem que constat speciem, alio modo per differentiam existentiam. ... Si fiat descensus per differentiam essentialem, tunc fit descensus in idem genus simpliciter, nam eadem scientia habet considerare de superiori et de suo per se inferiori. Eadem enim scientia que considerat de triangulo considerat de ysoceli. Si autem fiat descensus per differentiam existentiam, tunc non fit descensus in idem genus simpliciter, sed in idem genus quodammodo ut si per probata de linea descendenda ad probandum de alia de linea visuali. Tunc fit descensus in idem genus quodammodo, nam scientia subalternans et scientia subalternata sunt idem quodammodo. Isto duplici modo convenit descendere a genere in genus ut descendendo in idem genus simpliciter ut descendendo in idem genus quodammodo."

[23] See the references in note 4 above.

[24] The following account of Ockham's position on *metabasis* is discussed more fully in my *William of Ockham, the Subalternate Sciences, and Aristotle's Theory of* metabasis, forthcoming in British Journal for the History of Science.

In the prologue to his *Ordinatio*, which Ockham completed before he left for Avignon in 1324, Ockham discusses the nature of science and in particular, the ways by which one science interacts with another.[25] Ockham argues that *scientia* may be taken in two different senses. On the one hand, it may refer to a single proposition segregated from all others and hence autonomous. But, says Ockham, this is generally not what we mean when we speak of *scientia*, nor is it what Aristotle had intended when he discussed the sciences in the *Posterior Analytics* or the *libri naturales*. Rather, *scientia* is usually taken to mean a collection of propositions whose unity derives not from a single subject, but from the order which brings together these propositions. Under this conception of science, Ockham suggests that a proposition separated from its order may well apply to several other sciences. And most importantly, this idea of science leads him in turn to suggest that some sciences overlap and that Aristotle's prohibition of *metabasis* applied only to *scientia* when taken in the first sense: that is, the autonomy of single propositions. However, says Ockham, when the subject of one science is contained under the subject of another, it is entirely permissible for one science to demonstrate the conclusions of another. How does this happen? Clearly, Ockham recognized that Aristotle had excluded only the traditional subalternate sciences from his prohibition of *metabasis*, because at this point, he expands upon Aristotle's position:

> If it is said that the Philosopher excludes only the subalternate and subalternating sciences, ... it has to be said that by this he intended to exclude, but did not do so explicitly, certain other sciences which are subalternated in other ways.[26]

Now, in Part III of his *Summa Logicae*, Ockham elaborates on the several ways by which sciences are subalternated, drawing upon and making more explicit a notion of partial subordination that had been discussed as early as Robert Grosseteste a century before. One need not suppose that the higher mathematical science is completely extended to the subalternate sciences; rather, only part of it may extend to all or even part of the inferior sciences. More important still, Ockham argues that there is both a broad and strict sense of subalternation. Strictly speaking, sciences such as optics, astronomy, music, and the like are subalternated to arithmetic or geometry according to the criteria that Aristotle had suggested. But Ockham also suggests that

[25] *Scriptum in librum primum Sententiarum Ordinatio*, Prologue, q.1.; ed. Gedeon Gál and Stephen Brown, *Opera theologica*, vol. 1, St. Bonaventure, N.Y. 1967, 7-15.
[26] *Ordinatio*, Prologue, q.1.; *OTh* 1.14, 16-20.

there is a broad sense according to which sciences not normally seen as subalternating can be considered in that capacity. In spite of the fact that Aristotle had stated explicitly that medicine and geometry are not subalternate sciences, Ockham suggests that broadly speaking they are, and likewise, one may also consider logic or metaphysics and the special sciences in this same broad subalternating relationship.[27]

What is clear from this is that Ockham has altered Aristotle's prohibition of *metabasis* from two sides. On the one hand, by restricting the prohibition to only those sciences that are completely distinct in subject and properties—that is, Ockham's idea of science as a single proposition—Ockham has removed much of the force that the original theory possessed. And on the other hand, by greatly expanding the number of sciences that may be considered subalternating and subalternate, Ockham escapes the injunction of *metabasis* in most cases. If a science cannot qualify as subalternate under Aristotle's strict definition, it may well qualify under Ockham's more liberal rules.

Some ten years after Ockham's original discussion, Thomas Bradwardine takes up the problem of *metabasis* implicitly when he considers the generic incompatibility of mathematics and physics in part 4 of his second chapter to the *Tractatus de proportionibus*. Chapter 2 as a whole deals with what Bradwardine calls "four erroneous positions" regarding the relationship between velocity and the powers that cause motion. Already he has argued against three: that velocity is proportional to the difference between motive and resistive power, that it follows the proportion between that difference and the resistive power, and that it corresponds to the proportion between motive and resistive power.[28] All of these, Bradwardine notes, are similar inasmuch as they assume that there is a mathematical relationship between velocity, motive power, and resistive power. But in the fourth erroneous position, Bradwardine considers the objection that natural philosophy and mathematics are generically different and that therefore there is no mathematical relationship between motive and resistive powers. Brad-

[27] *Summa logicae* III, ii, ch. 20-21; ed. Gedeon Gál and Stephen Brown, *Opera philosophica*, vol. 1, (St. Bonaventure, N.Y. 1974), 537-542, esp. 539-541. Ockham is by no means unique in his ideas about partial subalternation; as I have suggested in my article on Ockham's theory of *metabasis*, Burley and others seem to be drinking from the same stream.

[28] *Thomas of Bradwardine, his Tractatus de proportionibus; its significance for the development of mathematical physics*, ed. and trans. H. Lamar Crosby, Jr., Madison, Wisc. 1955, 86-105.

wardine's reply centers on a distinction of two senses of proportion: in a strict sense, proportion refers only to quantities and therefore is restricted to scientific genera that deal with quantity. In a more general sense, however, proportion is found in all things which admit of being equal, greater or less, similar, or larger and smaller. And this, says Bradwardine, is the sense in which he uses proportion in the present context.[29] With varying degrees of success, Bradwardine challenges his fourth opponent, suggesting that he has conflated these two senses of proportion. But when the dust clears, we find that Bradwardine's conviction about the applicability of proportion to the topic of motion rests on his observations about the subalternate and subalternating sciences:

> "...if there were no proportion between powers because they are not quantities (of the same genus)," says Bradwardine, "by the same position, there could be no proportion between tones. And then the entire science of harmonics would be destroyed."[30]

Simple pragmatism on the one hand and I would suggest an expanded view of subalternation—as we have seen in Ockham—on the other seem to have exerted an influence on Bradwardine's view of *metabasis.*

If Bradwardine's extensive discussion of the issue indicates that he was concerned about the legitimacy of applying such proportions, scholars of the succeeding generation seem to have merely assumed it at the outset. Many of them had learned of Bradwardine's work not from the original *Tractatus de proportionibus*, but rather from abridgements of the work. And it is interesting and perhaps indicative of the concerns of the new generation that one somewhat popular version—the so-called *Tractatus brevis proportionum*—glosses over the argument of chapter 2, part 4, and instead moves on to part 5—Bradwardine's own function—to deduce several conclusions that follow from it.[31] In the same way, near the beginning of his *Regule solvendi*

[29] *Tractatus* 66, lines 1-10; 108-110, lines 481-516.

[30] *Tractatus* 106, lines 445-447; my translation.

[31] I am currently investigating the dissemination of Bradwardine's material through the agency of these abridged editions. Most are similar to the *Tractatus brevis proportionum: abbreviatus ex libro de proportionibus*, Vienna 1515, reproduced by Clagett in *Science of Mechanics in the Middle Ages*, Madison, Wisc. 1959; the argument of part II, chapter 4 has been reduced to "Quarta opinio ponit quod velocitas motus non sequatur proportionem, quia nulla est proportio inter potentiam et resistentiam. Sed omnia argumenta pro illa opinione solum procedunt de proportione proprie dicta. Ideo contra eam non arguo ad praesens." (p. 490). The version found in Vienna, Österreichische Nationalbibliothek 4784, fol. 231^{r}-236^{v}, is even more brief (fol. 236^{r-v}): "Quarta opinio est quod velocitas motuum non debet attendi penes aliquam proportionem. Similiter

sophismata, William Heytesbury remarks that he will assume in the course of his work that "any finite latitude is a certain quantity."[32] And at the beginning of treatise XIV of the *Liber calculationum*, Richard Swineshead points out that his treatment of local motion will proceed under the assumption that motion is measured according to geometrical proportion.[33] If the application of proportion and the legitimacy of quantification rested originally on an expanded version of subalternation, it would seem that the issues and assumptions of the generation of the 30s and 40s had shifted away from *metabasis* and instead concentrated on what have come to be called calculatory techniques.

In spite of this shift in emphasis, there would seem to be at least two aspects of Mertonian science which both facilitated the use of mathematics in natural philosophy and relieved concerns about the appropriate use of one science in the domain of another. Each treads upon the development in ideas about the subalternation of the sciences. When Aristotle prohibited the use of *metabasis* in the sciences, he made it clear that he was not concerned with the transference of general principles and axioms from one science to another, but rather with what he considered the misapplication of principles that are specific to one science. From Grosseteste onward, virtually every medieval commentator on the *Posterior Analytics* had noted that the principle "Equals taken from equals yield equals" is common to several sciences, but the principle "Equal magnitudes taken from equal magnitudes yield equal magnitudes" is specific to geometry and cannot be transferred to another science.[34] It is the latter, more specific principle that is constrained by the prohibition of *metabasis*.

One of the most striking features of fourteenth-century calculatory science is the extent to which logic enters the topics under discussion. This is perhaps nowhere more apparent than in Heytesbury's *Regule*. According to its prologue, the *Regule* was intended as a guide for first-

talis non stat." Still another version (Cambridge, Gonville and Caius 182/215, pp. 119-131), although more complete in other sections of the treatise, omits this portion of the text entirely.

[32] *Hentisberi de sensu composito et diviso, Regule solvendi sophismata* ..., Venice 1494, fol. 40va: "Quelibet latitudo finita est quedam quantitas."

[33] *Liber calculationum*, Venice 1520, fol. 43va: "...quedam regule de motu locali supponendo motum attendi proportionem geometricam."

[34] Robert Grosseteste, *Commentarius in Posteriorum Analyticorum libros*, ed. Pietro Rossi, Florence 1981, 137.

year students in logic.[35] What precisely is the relationship of logic to the mathematical or physical topics under discussion? Over the past thirty years, scholars have recognized a two-fold, reciprocating tie. On the one hand, according to Curtis Wilson, while mathematical and physical principles are used in Heytesbury's text, they are brought to bear on logical or semantic problems such as the problem of denomination; that is, under what conditions can a subject be said to be such-and-such? If Brownie the donkey begins to move and increases her speed, at what degree of velocity may we say she runs? Or if Socrates is lifting weights and reaches the limits of his strength, is there a maximum weight which he can lift, or a minimum weight which he cannot lift?[36] While problems such as these admit of mathematical or physical solutions, and indeed Heytesbury uses both in his discussions, he marshals these techniques in the solution of a logical or semantic problem.

More recently, however, John Murdoch has noted that the reciprocal relationship is also true of Mertonian and more generally, fourteenth-century work. By using what Murdoch calls "analytical languages," fourteenth-century scholars transferred problems in the object language to ones that were analyzed in a metalanguage; that is, rather than an analysis of problems the objects of which are things, those problems were translated in such a way that the objects became propositions, and by suitable reductions, operations on terms or propositions gave solutions—or in the case of *sophismata*, distinguished between proper and improper procedures—first in the metalanguage and then by reversal in the object language.[37]

[35] *Regule* fol. 4^{v}.

[36] *Regule* fol. 93^{r}ff., 141^{r-v}. See also Curtis Wilson (note 1 above) 21-25 and his *William Heytesbury*, in: *Dictionary of Scientific Biography*, New York 1972, vol. 6, 376-380.

[37] This aspect of fourteenth-century science has been emphasized by Murdoch in several of his more recent articles, but see in particular *The Analytic Character of Late Medieval Learning* (note 14 above), *The Development of a Critical Temper: New Approaches and Modes of Analysis in Fourteenth-Century Philosophy, Science, and Theology*, in: *Medieval and Renaissance Studies*, ed. Siegfried Wenzel, Chapel Hill, NC 1978, 51-79, and *Mathematics and Sophisms in Late Medieval Natural Philosophy*, in: *Les genres littéraires dans les sources théologiques et philosophiques médiévales: Définition, critique et exploitation*. Actes du colloque international de Louvain-la-Neuve 25-27 mai 1981, Louvain-la-Neuve 1982, 85-100. Murdoch notes that while fourteenth-century scholars used logic, it would be misleading to suggest that they are simply doing logic; "they are doing physics or natural philosophy but with decidedly logical, metalinguistic, tools" (*Analytic Character*, p. 196). See also Edith Sylla's discussion of this issue in *The Oxford Calculators*, in: *The Cambridge History of Later Medieval Philosophy*, ed. Norman Kretzmann, Anthony Kenny, and Jan Pinborg, Cambridge, 1982, 540-563 at 546-547.

This relationship between logic and the Mertonian work is not, I would suggest, an insignificant one, for in his discussion of *metabasis*, Aristotle had singled out dialectic as a peculiar science in the sense that unlike the special sciences—those of mathematics and natural philosophy—dialectic has no specific domain or subject matter on which it operates. This feature of dialectic lay at the foundation of Aristotle's prohibition of *metabasis*, for unlike Plato, Aristotle was far less convinced about the certainty of dialectical method in all areas of science, and it was against Plato's universal dialectic that Aristotle developed his theory of demonstration. Dialectic could proceed from knowledge based on belief or opinions; demonstration by contrast rested only on necessary principles. Dialectic must be prepared to argue a conclusion from the indeterminate response to a question; demonstration may respond to questions, but they must be questions that admit only one answer. Dialectic ranges over all fields; demonstration, bound by the prohibition of *metabasis*, is restricted to a single subject genus.[38]

One means of avoiding this prohibition, as I have already suggested, is to change the nature of the subject genus or expand the allowable exceptions, both of which can be seen in Ockham. But another, perhaps less obvious solution is to draw upon Aristotle's distinction between dialectic and demonstration. While Aristotle clearly favored demonstration as the method of the sciences, he did not dispose completely of the dialectical method. Instead he reserved it to the *Topics*, where it served as a method for discovering arguments.[39]

[38] Concerning the ultimate bases of dialectic, see *Topics* I.1 $100^{a}18$-21, $100^{b}22$-23 and *Posterior Analytics* I.19 $81^{b}19$-21. In *Topics* VIII.1 $155^{b}8$-17, Aristotle compares the method of the dialectician with that of the (demonstrative) philosopher. As for the subject genera of dialectic, Aristotle notes in *Sophistical Refutations* XI $172^{a}14$-17 that "...the art of examination is not knowledge of any definite subject, and it therefore follows that it deals with every subject; for all the arts employ also certain common principles." Aristotle repeats this in *Posterior Analytics* I.11 $77^{a}26$-35, and in *On the Generation of Animals* II.8 $747^{b}27$-30, he says that dialectical proof is more universal "the further removed it is from the special and proper principles." Likewise, in *Rhetoric* I.1, I.2, and I.4, Aristotle states that rhetoric, like dialectic, has no special genus for its subject matter. Compare this characterization with Aristotle's requirements for demonstration (*Posterior Analytics* I.7 $75^{a}38$ff.), which include the application of an essential attribute to some genus and the specification of the genus itself.

[39] Concerning Aristotle and the *Topics*, see Walter de Pater, *Les Topiques d'Aristote et la dialectique platonicienne; La méthodologie de la définition*, Fribourg 1965, and *Aristotle on Dialectic. The Topics. Proceedings of the Third Symposium Aristotelicum*, ed. G. E. L. Owen, Oxford 1968, esp. the articles by De Pater and Düring; and J. D. G. Evans, *Aristotle's Concept of Dialectic*, Cambridge 1977.

Now the fate of this topical method has been much-discussed by scholars over the past generation.[40] In the sixth century, Boethius held much the same view of dialectic as a method of discovering arguments and made the method more precise in his *De topicis differentiis*. But beginning in the eleventh century and continuing through the thirteenth century, we find a deviation from the traditional Aristotelian notion of topics. There is a growing sense that all syllogisms are dependent upon topics and that topical arguments as well as demonstrative ones are necessary. The result of this shift is first that topics become a method not as much for discovering as confirming arguments, and second—and for our purposes more important—a gradual blurring of the distinction between dialectic and demonstration.[41]

The significance of this development for the parallel problem of *metabasis* is considerable. The revaluation of dialectic, particularly in the new forms of scholastic logic like the *consequentiae*, brings with it the recognition that dialectic, and to some extent demonstration, may be free from Aristotle's remarks about *metabasis*. When coupled with the broad sense of subalternation we have seen in Ockham, this special property of logic becomes a significant one for Mertonian science. It is perhaps even more suggestive when we observe that in the third part of the *Summa Logicae*, Ockham had suggested that one science may in certain cases be both subalternating and subalternate with respect to different parts of another science.[42] In other words, logic as a science in its own right can both come to the aid of and be aided by mathematics and natural philosophy. If logic is subalternated and subalternating to both mathematics and natural philosophy, both sciences may be applied side-by-side without strictly speaking resorting to *metabasis*. Logic thus becomes a bridge between mathematics and physics.

Closely related to this bridge is another aspect of Mertonian science that bears upon quantification and *metabasis*. It has often been re-

[40] Eleanor Stump, *Topics: Their Development and Absorption into Consequences*, in: *The Cambridge History of Later Medieval Philosophy*, 273-299 and *Dialectic*, in: *The Seven Liberal Arts in the Middle Ages*, ed. David L. Wagner, Bloomington, Ind. 1983, 125-146.

[41] See, for example, Ockham's discussion at *Summa logicae* III, i, ch. 1; *OPh* 1.360 and III, ii, ch. 9; *OPh* 1.521 and Eileen Serene's discussion in *Demonstrative Science*, in: *The Cambridge History of Later Medieval Philosophy*, 496-517 at 514.

[42] *Summa Logicae* III, ii, ch. 20-21; *OPh* 1.537-542. See also the discussion in my article cited in note 24 above.

marked that Mertonian science is an analysis of problems *secundum imaginationem.* The problems Heytesbury considers, the solutions he proposes, and the objections to his position that he refutes are more likely than not framed according to imagination; empirical considerations and actual physical measurements are absent from his work.[43] Likewise, when Swineshead discusses the case of a very thin rod passing down a shaft to the center of the earth and asks whether it will reach the center of the universe, his problem is framed *secundum imaginationem.*[44] It matters little to Swineshead that this problem may be unverifiable or that the situation may be actually impossible. It is a problem framed *secundum imaginationem*, whose purpose is to test the boundary conditions of a theory under consideration, not determine how things actually behave in reality. Several years ago, A. G. Molland drew attention to the fact that by the early fourteenth century, *imaginatio* was closely associated with mathematics. Thomas Aquinas perceived mathematics as a science which receives its subject matter through the imagination, while physics does so through the senses. Henry of Ghent likewise suggested that those whose imaginative faculties were well-developed made the best mathematicians, the worst metaphysicians, and inept natural philosophers. And when Thomas Bradwardine discusses the mathematical properties of relationships between infinite classes in the *De causa Dei*, he does so *per imaginationem.*[45] Given this relationship between the medieval conception of *imaginatio* and the mathematical disciplines, are we led once

[43] Curtis Wilson (note 1 above) 24-25, 148-149, and 174 n. 65.

[44] *Liber calculationum* (note 33 above) fol. 35^{v}-37^{r}. The text has also been edited by M. A. Hoskin and A. G. Molland, *Swineshead on Falling Bodies: An Example of Fourteenth-Century Physics*, in: British Journal for the History of Science, 3 (1966), 150-182. On the same topic, see also Molland, *Richard Swineshead and Continuously Varying Quantities*, in: *Actes du XII^{e} congrès international d'histoire des sciences*, 4 (Paris 1968), 127-130.

[45] Molland (note 1 above) 113-114. Thomas Aquinas, *Expositio super librum Boethii de Trinitate* q. 5, a. 3 resp.; *Expositio in XII libros Metaphysicorum* VII, lect. 10, nn. 1494-1496. Henry of Ghent, *Quodlibeta*, Paris 1518, II, q. 9; fol. 36^{r}. Bradwardine, *De causa Dei* I, ch. 1, coroll. pars 40; ed. Henry Savile, London 1618, 121-124. Concerning the medieval theory of the imagination, see M. W. Bundy, *The Theory of Imagination in Classical and Medieval Thought*, Urbana, Ill. 1927. The faculty of the imagination and its importance for medieval perceptions of Aristotle's prohibition of *metabasis* has not generally been recognized. As the faculty that mediated between sense and reason first by recreating images received originally through the senses and then combining them to form new complexes, *imaginatio* itself was a kind of psychological *metabasis* or transition between dissimilar realms. This has been suggested recently for the case of Galilean science by Lorraine J. Daston, in: *Galilean analogies: Imagination at the Bounds of Sense*, in: *Isis*, 75 (1984), 302-310.

again to the middle sciences as the vehicle for eluding Aristotle's prohibition of *metabasis*? Unfortunately, the evidence on this score is somewhat mixed. As John Murdoch and Edith Sylla have noted,[46] works involving *calculationes* in early manuscripts were not grouped with mathematical texts like the *Elements*, but instead with treatises on logic or natural philosophy. This, of course, would seem to be quite understandable, given the logical component of the work and the fact, as Murdoch and Sylla emphasize, that medievals often tended to blur the distinction between mathematics and logic, moving freely from one to another. Furthermore, and perhaps more fundamentally, we should remember that the Arts Faculty at Oxford or any other medieval university had no provision for studying *calculationes* apart from the traditional curriculum of logic, mathematics, natural philosophy, ethics, and metaphysics. When such curricula are specified, mathematical work centered around such texts as the works of Boethius, Euclid's *Elements*, and Jordanus de Nemore's *Arithmetica*,[47] and while Bradwardine's theory of ratios has its roots in the *Elements*,[48] it seems clear from the manuscript evidence that medievals did not consider the mathematics of the *calculatores* congruent with that required in the arts faculties. Hence it should not be surprising that *calculationes* should be grouped by default among the logical or natural philosophical works.

It is perhaps more surprising that treatises dealing with the latitude of forms were grouped in codices with texts of more traditional *scientiae mediae* only beginning in the fifteenth century.[49] There are, however, some suggestions already in the fourteenth century that this conception was beginning to change. Given the link between calculatory tradition and *imaginatio*, one should note that as early as the beginning of the fourteenth century, Peter Auriol—whose work on the nature of science seems to have influenced Ockham's ideas on the same subject—characterizes the *scientiae mediae* as those disciplines in which

[46] John E. Murdoch and Edith D. Sylla, *The Science of Motion*, in: *Science in the Middle Ages*, ed. David C. Lindberg, Chicago 1978, 206-264 at 247-248. My own cursory review of the manuscripts of Heytesbury's *Regule* and Swineshead's *Liber calculationum* would suggest that quite often these texts were grouped with others of the same genre.
[47] James A. Weisheipl, *The Curriculum* (note 15 above) esp. 170-176. See also Edith Sylla's remarks in *The Oxford Calculators* (note 37 above) esp. 542-544.
[48] A. G. Molland, *An Examination of Bradwardine's Geometry*, in: Archive for History of Exact Sciences, 19 (1978), 113-175 and Sylla (note 37 above) 553-554.
[49] Murdoch and Sylla (note 46 above) 263 n. 137.

sense and *imaginatio* are joined.[50] Similarly, Henry of Langenstein already in the 1380s includes treatises on the latitude of forms not among the *scientiae sermocinales* or the disciplines of natural philosophy in his *arbor scientiarum*, but instead among the *scientiae mediae*.[51] What this may suggest, in other words, is the genesis of a wider conception of the middle sciences that becomes incorporated fully only in fifteenth-century texts and codices.

Historians of science have long pointed to the *scientiae mediae* as the mechanism for effecting a mathematical physics out of Aristotelian natural philosophy. Olaf Pedersen, for example, sees the creation of the concept of *scientiae mediae* by Aquinas as "un chemin du quadrivium à la physique."[52] And more recently, Jean Gagné has ex-

[50] Peter Auriol, *Scriptum super primum Sententiarum*, ed. Eligius M. Buytaert. St. Bonaventure, NY 1952, Prologue, q. 5, a. 2, no. 49 (p. 300). Concerning the relationship between Auriol and Ockham, see Paul Vincent Spade, *The Unity of a Science According to Peter Auriol*, in: Franciscan Studies, 32 (1972), 203-217.

[51] Henry of Langenstein, *Expositio prologi Bibliae*; MS Vienna, Österreichische Nationalbibliothek, CVP 3900, fol. 54^{ra-va}. The text is quoted fully by Nicholas H. Steneck, *A Late Medieval Arbor Scientiarum*, in: *Speculum*, 50 (1975), 245-269 at 250-251. Steneck has also noted that Henry argues against disciplinary autonomy elsewhere in his works; see *Science and Creation in the Middle Ages: Henry of Langenstein (d. 1397) on Genesis*, Notre Dame 1976, 145 and 191 n. 24. Henry's position, according to Steneck, bears the "unfortunate consequence ... that the study of nature has no rules that are specifically and exclusively applicable to it alone" (p. 145), and he therefore distinguishes Henry's science and medieval science more generally from modern science. In the general sense, of course, this is true; but seen against the background of Aristotle's prohibition of *metabasis*, Henry's statements, like those of other fourteenth-century scholars suggested in this paper, constitute an important hurdle toward modern methodology.

Earlier in this paper, we suggested that Bradwardine's argument in chapter II, part 4 of the *Tractatus* rested in part on the nature of music. Bradwardine's argument is likewise mirrored in Franciscus de Ferraria's *Questio de proportionibus motuum*, ed. Marshall Clagett in *Science of Mechanics* (note 31 above) at 501. The same argument seems to have appealed to Oresme in his discussion of intension and remission; see his *Quaestiones super geometriam Euclidis*, ed. Marshall Calgett in *Nicole Oresme and the Medieval Geometry of Qualities and Motions*, Madison, Wisc. 1968, q. 11 (pp. 546-547). The culmination of this link between the intermediate science of music and calculatory techniques may be found in the anonymous *Questiones musice* of Paris, BN lat. 7373 studied by John Murdoch [*Music and Natural Philosophy: Hitherto Unnoticed* Questiones *by Blasius of Parma*(?), in: Manuscripta, 20 (1976), 119-136]. Murdoch suggests that the text reflects the interests of the University of Pavia (p. 126), and if this is the case, it would also reflect late fourteenth-century views about the ties between a *scientia media* such as music and the new calculatory techniques, since Blasius taught there between 1379 and 1382, between 1389 and 1399 and again between 1403 and 1407.

[52] Olaf Pedersen, *Du quadrivium à la physique*, in: *Artes liberales von der antiken Bildung zur Wissenschaft des Mittelalters*, ed. Josef Koch, Leiden 1959, 107-123 at 123.

amined the development of the more traditional *scientiae mediae*, showing that among the Arabic and later Latin commentators, these intermediate sciences moved away from being tied to either physics or mathematics—or as Aristotle had specified, from being "the more physical parts of mathematics"—to become a *tertium quid*, truly intermediate between the two disciplines.[53] What has not generally been recognized is that later in the Middle Ages, not only was the position of the *scientiae mediae* beginning to change, but more fundamentally, the criteria by which a discipline could be considered subalternate or subalternating were changing as well. Sciences could be seen as partially subalternated in such a way that different parts could depend on distinct superior sciences for their principles. More important still, many more sciences could qualify for this relationship, and the new techniques of the fourteenth century were eventually incorporated in the body of the sciences under this relationship. And with this change, medievals were able to modify Aristotle's prohibition of *metabasis* that had precluded cross-disciplinary work in the sciences.

Somewhat more generally, this shift in the nature of the subalternate sciences and the attendant revision of Aristotle's theory of *metabasis* emphasizes an important aspect of scientific change in the Middle Ages. By and large, historians have adopted the view that while medievals modified ancient—and in particular Aristotelian—theories of natural philosophy, they did not usually challenge the basic assumptions or the core elements[54] of Aristotelian science. Bradwardine or Buridan, for example, may have adjusted the description of motion or the causes of projectile motion, but they left undisturbed the idea that every mobile requires a mover. Similarly, medieval scholars beginning with Grosseteste gave more precision to the distinction between *propter quid* and *quia* science, but underlying this work remained the conviction that *scientia* is a causal enterprise.[55]

[53] Jean Gagné, *Du quadrivium aux scientiae mediae*, in: *Arts libéraux et philosophie au moyen âge*. Actes du IV[e] congrès international de philosophie médiévale, Montreal-Paris 1969, 975-986.

[54] Larry Laudan, *Progress and its Problems*, Berkeley 1977, 95ff.

[55] Anneliese Maier, *'Ergebnisse' der spätscholastischen Naturphilosophie*, in: *Ausgehendes Mittelalter*, Rome 1964-1977, vol. 1.425-457; James A. Weisheipl, *The Principle* Omne quod movetur ab alio movetur *in Medieval Physics*, in: Isis, 56 (1965), 26-45; Grosseteste's position on *scientia propter quid* and *quia* is discussed by A. C. Crombie, *Robert Grosseteste and the Origins of Experimental Science*, Oxford 1953, esp. ch. 5; later medieval discussions of the theory are reviewed by John Herman Randall, *The School of Padua and the Emergence of Modern Science*, Padua 1961.

In the modification of the prohibition of *metabasis*, however, we have what would seem to be a shift in one of the core elements of Aristotelian science, for the prohibition of *metabasis* reflected fundamental aspects of Aristotle's ontological and methodological positions. Yet even in this respect, when medievals shifted the boundaries of the sciences, they did so under the wings of Aristotle: Ockham, as we have seen, suggests that if subalternation is possible in cases not prescribed by Aristotle, it is not because he has deviated from the Philosopher's position, but rather because he has made explicit what was implicit in Aristotle's ideas. And somewhat later in the fourteenth century, when Albert of Saxony asks why Aristotle would bother prohibiting *metabasis* when so many sciences are related, his answer is that Aristotle had never intended to apply the prohibition to the subalternate sciences (which Albert, like Ockham, now considers more numerous), but rather only to those sciences that are clearly distinct.[56]

If, however, fourteenth-century scholars reconciled to their satisfaction their own modification of *metabasis* with Aristotle's position, their solution was not unanimously held or at least permanent. Looking back from the early sixteenth century, Pietro Pomponazzi complained that Swineshead and the rest of the English calculators had put too much mathematics and geometry into natural philosophy. According to him, such sciences are illegal innovations between mathematics and physics.[57] Likewise, later in the sixteenth century, Francesco Buonamici, who was one of Galileo's teachers at Pisa, expresses his exasperation at "many of these (Latins) who leap from physics to mathematics."[58] And as Henry Guerlac has recently noted, as late as

[56] Albert of Saxony, *Quaestiones super libros Posteriorum*, Venice 1497, fol. D_1^{rb}: "Sed diceret aliquis ex quo sunt plures scilicet modi quibus convenit descendere de una scientia in aliam, quare Aristoteles pluries dicit non convenit demonstrantem descendere de genere in genus? Respondetur quod hoc non dicit sicut prius dicebatur et etiam non descendere convenit etc., eo modo quo passio probata de aliquo subiecto in scientia subalternata etiam probaretur de subiecto subalternante ... quando sunt duo subiecta diversarum scientiarum, et uni illorum inest aliqua passio per se et alteri illorum inest illa passio per accidens, tunc non convenit fieri descensum de genere in genus, illo modo quod passio demonstrata de subiecto cui inest per se et demonstraretur de subiecto cui inest per accidens. Propter hoc quod plerumque et turpes sunt passiones que insunt per accidens illi linee, geometra non probat quod linea recta sit linearum pulcherrima..."

[57] *De intensione et remissione formarum ac de parvitate et magnitudine*, Venice 1525, fol. 6^{va}, 9^{va-vb}. See also Curtis Wilson, *Pomponazzi's Criticism of Calculator*, in: Isis, 44 (1953) 355-362 at 357 and 360.

[58] Francesco Buonamici, *De motu libri X*, Florence 1591, 525. At p. 528, Buonamici considers the problem of whether proportions that operate in mathematics are applied

the end of the seventeenth century, the early French reviews of Newton's *Principia* focused their attention on—among other things—the fact that while this new work may have been a mathematical one, it was clearly not natural philosophy.[59] The problem of *metabasis* and the idea of disciplinary boundaries on which it was founded did not evaporate in the Middle Ages. Rather, *metabasis* and its avoidance was still as much an issue in the seventeenth century as it had been in the fourteenth.

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legitimately to the case of things in matter—the same concern Bradwardine dismisses in his *Tractatus* Chapter II, part 4. Concerning Buonamici's treatment, see Christopher Lewis, *The Merton Tradition and Kinematics in Late Sixteenth- and Early Seventeenth-Century Italy*, Padua 1980, 127-169.

[59] Henry Guerlac, *Newton on the Continent*, Ithaca, NY 1981, 50-51. It is perhaps notable that the reviewer in the Journal des sçavans (1688) regarded the *Principia* as a work in the mixed mathematical tradition of mechanics rather than natural philosophy.